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SYSTEM MODELLING UNDER ADDITIVE CHANGES

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System performance is degraded if random failures are not properly detected. In this paper a stochastic model is proposed which provides explicit information about the behaviour of linear, discrete stochastic systems under the influence of sudden failures which are additive functions to the failed parameter. The general theory is developed first, followed by an example for the case of additional plant noise. Possible utilizations of this model in the problem of detection are also discussed.

1. INTRODUCTION

The problem of failure detection and identification (FDI) is a key aspect of many real control applications. This is because physical systems are often subjected to unexpected changes, such as component failures and variations in operating conditions due to internal or external changes, that degrade the overall system performance. We may refer to such changes as "failures", although they may not represent actual failing of physical components.

An error in the initial estimation of a system parameter may also be viewed as such a failure. In order to maintain a high level performance, it is important that failures be promptly dedected and identified so that appropriate remedies can be applied. Since the notions of failure detection and identification (FDI) are applicable to almost any physical system that has been modelled accordingly, there have been many attempts and different approaches to the problem, such as: voting techniques [3], multiple hypothesis filter-detectors [1], [2], [11], jump process formulations [4], failure sensitive filters [8], innovations based detection systems [13], [17], [19], GLR (generalised likelihood ratio) tests [19], [20], functional redundancy [15], analytical redundancy and robustness [5], [12], linear quadratic methods [15], [16]. For a detailed characterization of these

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methods one is referred to the two excellent reviews of Willsky [16] and Isermann [6]. Applications on which FDI methods are tested come from a variety of scientific areas and include space technology, aerodynamics, medicine, economics and traffic control.

The feasibility and complexity of FDI schemes depends on the nature of the failure. Complete malfunction is often straightforward to detect, but detection of subtle changes that lead to system performance degradation often presents a more complex problem. The work in this paper is based on the author's Ph.D. thesis [13]. The applicability of the tests is extended here to failure types not so far researched. These include step biases and parametric faults in both the state and measurement equations. To overcome the problems of complexity and a-priori hypothesis of failure modes, as mentioned in [15], the decision process is splitted into two phases. In phase 1, detection and partial isolation of the failure using statistical methods is performed, while in phase 2 complete isolation and failure size estimation are carried out using GLR or other schemes.

2. MODELLING RANDOM CHANGES

The class of models considered is defined by the set of the following time-invariant, scalar difference equations,

$$x(k+1) = \varphi x(k) + w(k) \tag{1}$$

$$y(k) = \eta x(k) + v(k) \tag{2}$$

with the assumptions that x(0), w(i) and v(i) are white guassian sequences, uncorrelated with each other for all i and distributed normally $N(\bar{x},p(0))$, N(0,q) and N(0,r) respectively. Relations (1) and (2) are termed "state" or "plant" and "measurement" or "observation" equations. No loss of generality results from the absence of a deterministic control in (1) since the error in state estimation is independent of such an input [7].

The model so defined is not a unique representation of stochastic linear system behaviour. In general, appropriate state or measurement models may be either continuous or discrete for specific applications. Linear systems which are continuous in the state, may, however, after discretization, be represented by equation (1), and the discrete modelling of measurements received at an approximately constant update rate is approporiate to a wide range of practical situations.

Although the FDI schemes to be described are principally related to the scalar model, extensions to the general multivariable case are straightforward and will be discussed in a future paper. The treatment of systems with time varying parameters is also possible.

The noisy measurement sequence $\{y(k)\}$ is the sole source of information from the actual system regarding the system state x(k). Since accurate knowledge of x(k) is a prerequisite for precise system operation through feedback controls, a state estimator which operates on the measurement sequence is often used in practical applications. The estimator which minimizes a wide class of loss functions is the well known Kalman filter defined by the set of difference equations,

$$\hat{x}(k/k-1) = \varphi\,\hat{x}(k-1/k-1) \quad (3)$$

$$p(k/k-1) = \varphi^2\, p(k-1/k-1) + q \quad (4)$$

$$K(k) = p(k/k-1)\,\eta\,[\eta^2\, p(k/k-1) + r]^{-1} \quad (5)$$

$$\gamma(k) = y(k) - \eta\,\hat{x}(k/k-1) \quad (6)$$

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)\,\gamma(k) \quad (7)$$

$$p(k/k) = (1 - K(k)\,\eta)\,p(k/k-1) \quad (8)$$

It is also well known that,

$$\hat{x}(k/k) = E\,[x(k) \mid y^k] \quad (9)$$

$$E\,[\hat{x}(k/k)] = E\,[x(k)] \quad (10)$$

A change in the assumed value of a parameter of the model described by (1)–(2) will be considered as a failure. Such a failure may occur as a result of malfunction or as a result of wrong initial calculation of the value of the parameter. It may also be the result of the natural behaviour of the model. Specifically, the following cases may be considered, using the theory to be developed:

(a). Change in the state noise mean (assumed zero).

(b). Change in φ.

(c). Additional plant noise.

(d). Change in the measurement noise mean (assumed zero).

(e). Change in η.

(f). Additional measurement noise.

The following assumptions on the system model are made:

1. The system is stable, i.e. $|\varphi| < 1$.

2. The system is uniformly completely controllable and uniformly completely observable so that asymptotic stability of the filter is ensured.

The FDI scheme is required to perform the following operations:

1. Detection of failure occurence, which simply consists of making a binary decision: either a failure has occured or not.

2. Isolation of a failure, which refers to the problem of determining the source of the failure where more than one parameter is subject to change.

3. Estimation of time of failure occurence and its magnitude.

4. Reorganization of system model on the basis of 1–3, which entails reinitialization of model and filter parameters.

Failures are assumed to be single, i.e. no more than one failure may occur at a time, and also that they may occur with equal probability at any time instant. The size of the failure is also arbitrary but may be bounded above and/or below from physical considerations.

Given a system parameter p and a failure modelled by $h(k,\theta,v)$ then the value of the parameter following a failure is given by:

$$P_{new} = P_{old} + h(k, \theta, v) \quad (11)$$

where

$$v \in [v_l, v_u]$$

is the size of the failure constrained below by v_l and above by v_u, and

$$\theta \in [0, \infty]$$

is the time of failure occurence which takes a finite integer value if a failure occurs and is infinite otherwise.

Failures may be classified into three types:

Type I: jump

Type II: step

Type III: ramp and higher order

Following Willsky et al. [19], the three failure types may be modelled by the corresponding terms:

I. $v\delta_{k,\theta}$ II. $v\sigma_{k,\theta}$. III. $(a+kv)\,\sigma_{k,\theta}$ and $h\,(v,k)\,\sigma_{k,\theta}$

where $\delta_{k,\theta}$ is the Kronecker delta ($\delta_{k,\infty} = 0$), $\sigma_{k,\theta}$ is the unit step and h(v.k) is a polynomial in k. Type 1 models may be used for instantaneous failures of only one time unit duration. Type II models represent failure cases of constant size which have a permanent effect on the system. Since estimation of a failure is carried out simultaneously with,or following detection, it will be assumed that,

$$t_f > \max\ \{ t_d, t_e \}\ = t_e$$

where t_f denotes failure duration, t_d detection time and t_e estimation time. Under this assumption consecutive steps could be monitored, since the failure monitoring process would have detected, estimated and subsequently reinitialised the filter parameters before the occurrence of a new failure.

3. EFFECT OF FAILURES ON FILTER INNOVATIONS

3.1. Effect on filter equation

Given the observability conditions, the true system is observable through the measurement sequence $\{y(k)\}$ only and equations (3)–(8) imply that knowing the measurement residual sequence $\{\gamma(k)\}$ is equivalent to knowing $\{y(k)\}$. It therefore follows that $\{\gamma(k)\}$ will contain information of failures, provided that the failures are observable. Although $\{y(k)\}$ and $\{\gamma(k)\}$ both contain information of a failure, the use of $\{\gamma(k)\}$ for FDI purposes is fundamentally more attractive in a scheme based on statistical inference, since the residuals have been shown to be white, with zero mean, while the measurements do not possess these properties.

If no failure occurs, the residuals generated by the system measurements and the Kalman filter are of known distribution. It a failure occurs, the filter algorithm will operate on the assumed values, generating residuals which no longer belong to the expected distribution. By obtaining the probability distribution of the residuals generated by the Kalman filter after each type of failure, FDI can be performed by testing which of the possible probability distribution represent $\{\gamma(k)\}$. Estimation of the size of the failure can then be performed by estimating a parameter of the appropriate probability distribution,

It is shown in [20] that for a jump bias in the plant state modelled by,

$$x(k+1) = \varphi x(k) + w(k) + \delta_{k+1,\theta} v$$
$$y(k) = \eta x(k) + v(k)$$

the state, measurement, state estimate and innovations sequences may be expressed as,

$$x(k) = x_0(k) + \varphi^{k-\theta} v \tag{12}$$
$$y(k) = y_0(k) + \eta\varphi^{k-\theta} v \tag{13}$$
$$\hat{x}(k/k) = \hat{x}_0(k/k) + f(k,\theta)\, v \tag{14}$$
$$\gamma(k) = \gamma_0(k) + g(k,\theta)\, v \tag{15}$$

where $x_0(k)$, $y_o(k)$, $\hat{x}_0(k/)$ and $\gamma_0(k)$ represent the values of the corresponding variables that would be obtained if no failure occurs, and the additional terms exist if a failure occurs at time θ and are calculated from the recurrence relations,

$$g(k,\theta) = 0; \quad k < \theta \tag{16}$$
$$f(k,\theta) = 0; \quad k < \theta \tag{17}$$
$$g(\theta,\theta) = \eta \tag{18}$$
$$f(\theta,\theta) = K(k)\,\eta \tag{19}$$
$$g(k,\theta) = \eta\,[\varphi^{k-e} - \varphi f(k-1,\theta)]; \quad k > \theta \tag{20}$$
$$f(k,\theta) = K(k)\, g(k,\theta) + \varphi f(k-1,\theta); \quad k > \theta \tag{21}$$

An important feature of these relations is the fact that the Kalman filter residuals, as indeed all parameters directly affected by the occured failure, can be written as the sum of two terms, one of which models effects solely due to θ and v and the other represents all effects other than those due to θ and v. This result, obtained for the particular case of a jump bias in the plant state, may be generalised for any failure of the additive class, as the folowing theorem implies:

Theorem. The state, measurement, filter state estimate and innovations sequences for models represented by (1)–(2) and (3)–(11) which are subject to sudden failures modelled by any additive function, may be expressed as:

$$x(k) = x_0(k) + h_x(k,\theta,\Delta p) \tag{22}$$
$$y(k) = y_0(k) + h_y(k,\theta,\Delta p) \tag{23}$$
$$\hat{x}(k/k) = \hat{x}_0(k/k) + f(k,\theta,\Delta p) \tag{24}$$

$$\gamma(k) = \gamma_0(k) + g(k, \theta, \Delta p) \tag{25}$$

where,

$h_x(k, \theta, \Delta p)$ is the effect on state x(k) of a failure of size Δp which occured at time θ,

$h_y(k, \theta, \Delta p)$ is the corresponding effect on measurement y(k),

$f(k, \theta, \Delta p)$ is the effect on the state estimate $\hat{x}(k/k)$, and

$g(k, \theta, \Delta p)$ is the effect on the residual $\gamma(k)$.

Further, the recursions on h_x, h_y, f and g are given by:

$$g(k, \theta, \Delta p) = h_y(k, \theta, \Delta p) - \eta\varphi f(k-1, \theta, \Delta p) \tag{26}$$

$$f(k, \theta, \Delta p) = K(k) g(k, \theta, \Delta p) + \varphi f(k-1, \theta, \Delta p); k \geqslant \theta \tag{27}$$

$$g(k,\theta, \Delta p) = f(k, \theta, \Delta p) = 0; \quad k < \theta \tag{28}$$

(proof in appendix).

The quantities h_x and h_y depend on the particular failure type but in view of (1)–(2), if a failure occurs in a parameter of the plant equation,

$$h_x(k, \theta, \Delta p) \neq 0$$

and

$$h_y(k, \theta, \Delta p) = \eta h_x(k, \theta, \Delta p); \quad k \geqslant \theta$$

but if a failure occurs in a parameter of the measurement equation,

$$h_x(k, \theta, \Delta p) \equiv 0; \quad \text{all } k$$

$$h_y(k, \theta, \Delta p) \neq 0; \ k \geqslant \theta$$

If a failure does not occur, h_x and h_y are identically zero.

Eqns. (22)–(28) provide a model for the evolution of the $\{x(k)\}$, $\{y(k)\}$, $\{\hat{x}(k/k)\}$ and $\{\gamma(k)\}$. However, the state estimate and innovations sequences are still calculated by the Kalman filter using the real system measurements y(k) from the equations,

$$\gamma(k) = y(k) - \eta\varphi\,\hat{x}(k-1/k-1)$$

$$\hat{x}(k/k) = \varphi\hat{x}(k-1/k-1) + K(k)\,\gamma(k)$$

The modelling of the state estimate and innovations sequences by (24)–(25) is useful because it enables system performance to be analysed and checked under any failure condition that can be represented as an additional function, modelled by $h_y(k,\theta,\Delta p)$ in the measurements.

3.2. Effect of failures on the joint pdf of the innovations

Having established the form of the innovations sequence under general failure conditions, their joint probability distribution function (jpdf) will now be examined.

In normal operation the statistical properties of the residuals are fully described by a normal distribution with,

$$\bar{\gamma}(k) \triangleq E[\gamma(k)] = 0; \quad \text{all } k \tag{29}$$

$$c\ (k,m) \triangleq E[\gamma(k)\,\gamma(m)] = 0; \quad \text{all } k \neq m \tag{30}$$

$$c\ (k,k) \triangleq E[\gamma^2(k)] = \eta^2\ p\,(k/k-1) + r; \text{ all } k \tag{31}$$

When a failure occurs, the residuals generated by the Kalman filter evolve according to (25). Since the linear structure of the Kalman filter equations and state and measurement models, is not changed in the presence of an additive type of failure, the residuals remain a linear combination of the guassian measurement sequence $\{y(k)\}$, and are therefore also gaussian. This result implies that the joint pdf of the innovations will be completely characterised by its first and second moments. The effect of the failure on t' : whiteness property must be examined as well. In normal operation the whiteness property enables the jpdf of the residuals to be written as the product of the individual pdf's of each residual. If this property does not hold, an orthogonalisation procedure may be employed.

To ease notational complexity the following definitions are made:

$$\underline{\gamma}^{j,k} \triangleq [\gamma(j),\ \gamma(j+1), \ldots, \gamma(k)]^T \tag{32}$$

$$\bar{\underline{\gamma}}^{j,k} \triangleq E\left\{[\gamma(j),\ \gamma(j+1), \ldots, \gamma(k)]^T\right\} \tag{33}$$

$$= [E(\gamma(j)),\ E(\gamma(j+1)), \ldots, E(\gamma(k))]^T$$

$$C^{j,k} \triangleq \operatorname{cov}[\underline{\gamma}^{j,k},\ \underline{\gamma}^{j,k}] \tag{34}$$

$$= E\left\{[\underline{\gamma}^{j,k} - \bar{\underline{\gamma}}^{j,k}]\,[\underline{\gamma}^{j,k} - \bar{\underline{\gamma}}^{j,k}]^T\right\}$$

Using these definitions, the jpdf of the gaussian vector $\underline{\gamma}^{j,k}$ is:

$$P(\underline{\gamma}^{j,k}) = \frac{1}{(2\pi)^{1/2n}\,|C^{j,k}|^{1/2}} \exp\left\{-1/2[\underline{\gamma}^{j,k} - \bar{\underline{\gamma}}^{j,k}]^T [C^{j,k}]^{-1} [\underline{\gamma}^{j,k} - \bar{\underline{\gamma}}^{j,k}]\right\} \tag{35}$$

where $n = k-j+1$ is the dimension of the residual vector.

If a failure has not occured, (35) becomes:

$$P(\underline{\gamma}^{j,k}) = \prod_{m=j}^{k} \frac{1}{(2\pi c\,(m,m))^{1/2}} \exp\left\{-1/2\,\frac{\gamma^2(m)}{c\ (m,m)}\right\} \tag{36}$$

$$\triangleq \pi\ (j,k)$$

In the event of a failure, (25) gives

$$E\{\gamma(k)\} = E\{\gamma_0(k) + g(k, \theta, \Delta p)\}$$
$$= g(k, \theta, \Delta p) \tag{37a}$$

if the second term in the expectation is non-random.

Otherwise,

$$E\{\gamma(k)\} = E\{g(k, \theta, \Delta p)\} \tag{37b}$$

Therefore the residual mean vector is, in general:

$$\bar{\underline{\gamma}}^{j,k} = [0, 0, \ldots, E\{g(\theta, \theta, \Delta p)\}, \ldots, E\{g(k, \theta, \Delta p)\}]^T \tag{38}$$

The residual covariance matrix can be calculated considering,

$$\mathrm{cov}\{\gamma(k), \gamma(m)\} = E\{(\gamma(k) - \bar{\gamma}(k))(\gamma(m) - \bar{\gamma}(m))\} \tag{39}$$
$$= E\{[\gamma(k) - E(g(k, \theta, \Delta p))][\gamma(m) - E(g(m, \theta, \Delta p))]\}$$

Again, if g is non-random,

$$\gamma(k) - \bar{\gamma}(k) = \gamma_0(k) + g(k, \theta, \Delta p) - g(k, \theta, \Delta p)$$
$$= \gamma_0(k)$$

Hence, in this case,

$$\mathrm{cov}\{\gamma(k), \gamma(m)\} = 0; \quad k \neq m$$
$$= c(k, k); \quad k = m \tag{40a}$$

Otherwise the R.H.S. of (39) is calculated using (37b) as,

$$\mathrm{cov}\{\gamma(k), \gamma(m)\} = E\{(\gamma(k) - E(g(k, \theta, \Delta p))(\gamma(m) - E(g(m, \theta, \Delta p))\}$$
$$= E\{\gamma(k)\gamma(m)\} + E\{E[g(k, \theta, \Delta p)]\, E[g(m, \theta, \Delta p]\} -$$
$$- E\{E[g(k, \theta, \Delta p)]\gamma(m)\} - E\{\gamma(k)\, E[m, \theta, \Delta p)]\}$$

The expectations of the terms in braces, are:

$$E\{\gamma(k)\gamma(m)\} = E\{[\gamma_0(k) + g(k, \theta, \Delta p)][\gamma_0(m) + g(m, \theta, \Delta p)]\}$$
$$= E\{\gamma_0(k)\gamma_0(m)\} + E\{\gamma_0(k)\, g(m, \theta, \Delta p)\} +$$
$$+ E\{\gamma_0(m)\, g(k, \theta, \Delta p)\} + E\{g(k, \theta, \Delta p)\, g(m, \theta, \Delta p)\}$$
$$= c(k,m) + E\{g(k, \theta, \Delta p)\, g(m, \theta, \Delta p)\}$$

$$E\{E[g(k, \theta, \Delta p)]\gamma(m)\} = E\{g(k, \theta, \Delta p)\}\, E\{\gamma(m)\}$$
$$= E\{g(k, \theta, \Delta p)\}\, E\{\gamma_0(m) + g(m, \theta, \Delta p)\}$$
$$= E\{g(k, \theta, \Delta p)\}\, E\{g(m, \theta, \Delta p)\}$$

Finally,

$$\mathrm{cov}\{\gamma(k)\gamma(m)\} = c(k,m) + E\{g(k, \theta, \Delta p)\, g(m, \theta, \Delta p)\}$$
$$- E\{g(k, \theta, \Delta p)\}\, E\{g(m, \theta, \Delta p)\} \tag{40b}$$

Having calculated the mean and covariance functions of the jpdf of the residual sequence in the event of a failure, the jpdf can be derived by susbstituting these terms in (35).

3.3. An example

To elaborate on the above results a specific case which is characteristic of the FDI problem will be considered. Let us assume a situation where additional plant noise is introduced into the system.Then, (1) becomes:

$$x(k+1) = \varphi x(k) + w(k) + \zeta_x(k)\, \sigma_{k+1,\theta} \tag{1a}$$

where $\zeta_x(k)$ is conveniently defined as a white guassian random sequence, independent of x(0), w(i), v(i) for all i,k and of zero mean und unknown constant variance s_x. In this case (2) remains unchanged. Now, if $\theta = k+1$, (1a) becomes,

$$x(k+1) = x_0(k+1) + \zeta_x(k)$$

and at time k+2,

$$x(k+2) = x_0(k+2) + \zeta_x(k+1) + \varphi\zeta_x(k)$$

Therefore, in general,

$$h_x(k, \theta, \zeta_x) = \sum_{i=0}^{k} \varphi^{k-i}\zeta_x(i)$$

Hence, using (25), the residual sequence may be writen as:

$$\gamma(k) = \gamma_0(k) + g_c(k, \theta, \zeta_x)$$

$$= \gamma_0(k) + \sum_{i=0}^{k} g_c(k, i)\, \zeta_x(i) \tag{41}$$

where the g_c can be calculated iteratively using (26)–(27), as

$$g_c(i, j) = \eta\, [\varphi^{i-j} - \varphi\, f_c\, (i-1, j)\,]$$

$$f_c(i, j) = K(i)\, g_c\, (i, j) + \varphi f_c\, (i, j); \quad i \geqslant j$$

The expected value of the residuals is, using (41),

$$E\left\{\gamma(k)\right\} = E\left\{\gamma_0(k) + \sum_{i=0}^{k} g_c(k, i)\, \zeta_x(i)\right\}$$

$$= 0$$

since both γ and ζ are zero mean.

The covariance is given by,

$$\text{cov}\left\{\gamma(k), \gamma(m)\right\} = E\left\{ [\gamma_0(k) + \sum_{i=0}^{k} g_c(k, i)\, \zeta_x(i)\,]\, [\gamma_0(m) + \sum_{j=0}^{m} g_c(m, j)\, \zeta_x(j)\,]\right\}$$

Now, since $E\{\zeta_x(j)v(i)\}=0$ for all i, j and $E\{\zeta_x(i)\zeta_x(j)\}=0$ for all $i \neq j$, it follows that,

$$\operatorname{cov}\{\gamma(k),\gamma(m)\} = c(k,m) + \sum_{i=0}^{\lambda} g_c(k,i)\, g_c(m,i)\, s_x$$

where $\lambda = \min\{k,m\}$.

The residual covariance matrix is then given by:

$$C_c^{j,k} = \left[\begin{array}{c|c} C^{j,\theta-1} & 0 \\ \hline 0 & C_c^{\theta,k} \end{array}\right]$$

where,

$$C_c^{\theta,k} = \begin{bmatrix} c(\theta,\theta)+c_c(\theta,\theta) & c_c(\theta+1,\theta)+ & \cdots\cdots\cdots & c_c(\theta,k) \\ c_c(\theta+1,\theta) & c(\theta+1,\theta+1)+c_c(\theta+1,\theta+1) & \cdots & c_c(\theta+1,k) \\ \cdots & \cdots & \cdots & \cdots \\ c_c(\theta,k) & & & c_c(k,k)+c_c(k,k) \end{bmatrix}$$

and

$$c_c(i,j) = \sum_{m=0}^{\lambda} g_c(i,m)\, g_c(j,m)\, s_x \tag{42}$$

It can be seen from the form of (42) that the residual sequence following an increase in the plant noise is not stationary as well as not white, since in general,

$$c_c(i,j) \neq c_c(i+m,j+m)$$

However, in steady state following a fault, it may be shown that,

$$c_c(i,j) = c_c(i+m,j+m) = c_c(i-j)$$

$$= s^{i-j}\frac{\eta^2}{1-s^2}\, s_x \tag{43}$$

where

$$s \,\underline{\Delta}\, (1-K\eta)\varphi$$

The form of (43), implies that under filter stability conditions,

$$\lim_{(i-j)\to\infty} c_c(i-j) = 0$$

i.e. the correlation between residuals following an increase in plant noise decreases exponentially with the distance between them. In steady state following a fault, the covariance matrix may therefore be written:

$$C_c^{j,k} = \begin{bmatrix} c(j,j)+c_c(0) & c_c(1) \ldots\ldots\ldots\ldots & c_c(k) \\ c_c(1) & c(j+1,j+1)+c_c(0) \quad \ldots & c_c(k-1) \\ \ldots & \ldots & \ldots \\ c_c(k) & c_c(k-1) \quad \ldots\ldots\ldots & c(k,k)+c_c(0) \end{bmatrix}$$

The jpdf in this case is:

$$p(\underline{\gamma}^{j,k}) = \pi(j,\theta-1)\,\frac{1}{2\pi^{1/2(k-\theta+1)}|C_c^{\theta,k}|^{1/2}}\,\exp\left\{-1/2\,[\underline{\gamma}^{\theta,k}]^T[C_c^{\theta,k}]^{-1}\underline{\gamma}^{\theta,k}\right\}$$

4. FAULT MONITORING

The results obtained for the jpdf of residuals in the event of a failure occurence, lead quite naturally to a hypothesis testing formulation of the fault monitoring process. Thus, the hypothesis that the generated residuals belong to class C_0 (no failure) against the hypothesis that they belong to an alternative class C_i may be tested.

As was asserted previously, the FDI proccess must be designed in such a way as to be able to be applied in a wide range of practical situations with various requirements of cost and complexity. The knowledge of the effects of the individual faults on the Kalman filter innovations can be used to design a scheme that operates on two levels. The first level may be a simple fault detection mechanism which also performs partial isolation of the failed parameter. On the sounding of an alarm from this first level, the second mechanism is activated. This performs the functions of failure isolation, estimation of time of occurrence and size of fault and subsequent system reorganization. Furthermore, if requirements so dictate, the two levels can be used independently of each other.

5. CONCLUSIONS

In this paper an adaptive filtering technique for discrete, linear stochastic systems subject to sudden "failures" is developed. The technique is potentially useful in the design of failure detection and compensation systems.

The proposed system consists of an explicit model for the calculation of the joint probability density function of the innovations sequence of the Kalman-Bucy filter in the event of a failure, whose likelihood may be tested against a null hypothesis of no-failure.

APPENDIX

The proof will be by induction. Suppose (22)–(25) hold for time k. At k+1, $\hat{x}(k+1\,/\,k+1)$, $\gamma(k+1)$ are calculated by the Kalman filter as,

$$\begin{aligned}\gamma(k+1) &= y(k+1) - \eta\varphi\,\hat{x}(k/k)\\ &= y_0(k+1)+h_y(k+1,\theta,\Delta p) - \eta\varphi\left\{\hat{x}_0(k/k)+f(k,\theta,\Delta p)\right\}\\ &= \gamma_0(k+1)+h_y(k+1,\theta,\Delta p) - \eta\,\varphi\,f(k,\theta,\Delta p)\end{aligned} \quad (A1)$$

and

$$\begin{aligned}\hat{x}(k+1/k+1) &= \varphi\hat{x}(k/k)+K(k+1)\,\gamma(k+1)\\ &= \varphi\left\{\hat{x}_0(k/k)+f(k,\theta,\Delta p)\right\} +\\ &\quad +K(k+1)\left\{\gamma_0(k+1)+h_y(k+1,\theta,\Delta p)-\eta\varphi f(k,\theta,\Delta p)\right\}\\ &= \hat{x}_0(k+1/k+1)+\varphi f(k,\theta,\Delta p) +\\ &\quad +K(k+1)\left\{h_y(k+1,\theta,\Delta p)-\eta\varphi f(k,\theta,\Delta p)\right\}\end{aligned} \quad (A2)$$

where the subscript 0 denotes the value of the parameter that is obtained if no failure occurs. Equations (A1)–(A2) may be rewritten,

$$\gamma(k+1) = \gamma_0(k+1)+g(K+1,\theta,\Delta p)$$
$$\hat{x}(k+1/k+1) = \hat{x}(k+1/k+1)+f(k+1,\theta,\Delta p)$$

where,

$$g(k+1,\theta,\Delta p) = h_y(k+1,\theta,\Delta p) - \eta\varphi f(k,\theta,\Delta p)$$
$$f(k+1,\theta,\Delta p) = \varphi f(k,\theta,\Delta p)+K(k+1)\,g(K+1,\theta,\Delta p)$$

At $k=\theta$, since the fault has not affected $\hat{x}(\theta-1/\theta-1)$,

$$\begin{aligned}\gamma(\theta) &= y(\theta)-\eta\varphi\hat{x}(\theta-1/\theta-1)\\ &= y_0(\theta)+h_y(\theta\cdot\theta,\Delta p)-\eta\varphi\hat{x}(\theta-1/\theta-1)\\ &= \gamma_0(\theta)+h_y(\theta,\theta,\Delta p)\end{aligned}$$

and

$$\begin{aligned}\hat{x}(\theta,\theta) &= \hat{x}(\theta-1/\theta-1)+K(\theta)\,\gamma(\theta)\\ &= \hat{x}_0(\theta/\theta)+K(\theta)\,h_y(\theta,\theta,\Delta p)\end{aligned}$$

Hence,

$$\gamma(\theta) = \gamma_0(\theta)+g(\theta,\theta,\Delta p)$$
$$\hat{x}(\theta/\theta) = \hat{x}_0(\theta,\theta)+f(\theta,\theta,\Delta p)$$

where

$$g(\theta,\theta,\Delta p) = h_y(\theta,\theta,\Delta p)$$
$$f(\theta,\theta,\Delta p) = K(\theta)\,g(\theta,\theta,\Delta p)$$

This completes the proof.

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